Shortwave Transport in the Cloudy Atmosphere by Anomalous/Lévy Diffusion: New Diagnostics Using FORTÉ Lightning Data

A. B. Davis
Los Alamos National Laboratory
Space & Remote Sensing Sciences Group
Los Alamos, New Mexico

D. M. Suszcynsky
Los Alamos National Laboratory
Space & Atmospheric Sciences Group
Los Alamos, New Mexico

A. Marshak
National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

Introduction

Anomalous photon diffusion can be described as an ad hoc modification of the popular 2-stream approximation, specifically the δ-Eddington/diffusion version, for monochromatic radiative transfer in a scattering plane-parallel atmosphere. In the physical picture that describes the standard diffusion (hence the 2-stream) model, photons wander through horizontally infinite dense cloudy layers along convoluted random walks made of many short (theoretically Gaussian) steps. This is largely why they are often reflected rather than transmitted. In the modified diffusion model, three-dimensional (3-D) radiative transfer is mimicked by allowing the photons to also make the occasional large jump, presumably from cloud-to-cloud in a layer, between different cloud layers, or to/from the ground. These jumps are described by Lévy-stable distributions, which are parameterized by an exponent "α," varying from 1 to 2 that controls the slow decay in their power-law tails, the Gaussian model being retrieved by setting $\alpha = 2$. Davis and Marshak (1997) originally introduced the new model to explain the well-documented reduction of albedo in 3-D cloud structures for a given average optical depth, but other approaches such as the independent-column approximation can also do that (Cahalan 1989; Davis 1992; Barker 1996; Evans and Benner 1999; Davis and Marshak 2000a). The unique feature of the Lévy-diffusion model predicts a generalized relation between (mean) pathlength in transmission, the overall physical thickness of the cloud system, and (area-average) optical depth. Pfeilsticker (1999) exploited this relation to empirically validate the new diffusion model with solar-spectrum data using differential absorption spectroscopy in the O_2 A-band. We recently uncovered further evidence of the model's validity in time-resolved optical lightning data captured by a new United States Department of Energy (DOE) nonproliferation satellite called "FORTÉ" which stands for Fast On-orbit TRansient Experiment.

Interestingly, FORTÉ is in almost the exact reciprocal (source/detector symmetric) geometry of Pfeilsticker's ground-based observations. In the following pages, we present the new evidence and draw some far-reaching conclusions.

In the next section we survey the features of photon-diffusion and Lévy-flight theories important for this study in time-dependent atmospheric radiative transfer. The following sections present the FORTÉ mission with a brief description of the instrumentation and on-board processing. We survey the phenomenology of optical lightning transients at the source, and how they propagate through the necessarily cloudy atmosphere, as captured in FORTÉ's data stream. We also analyze the results from FORTÉ from the perspective of photon diffusion theory and show that the data is more consistent with the anomalous (Lévy-flight) theory than the standard (Gaussian) one. In the last section, we summarize our findings, outline future work, and make some recommendations.

From Photon Diffusion to Lévy-Flight Heuristics

Standard Diffusion Theory

Possibly the most well-known fact about radiative transfer is that the average number of scatterings $\langle n \rangle_T$ of light transmitted (subscript "T") through a medium of relatively large optical thickness τ goes as τ^2. This is generally recognized as a direct consequence of photon diffusion (random walk) theory, as captured by Einstein's law for Brownian motion:

$$\text{Mean squared distance from source, } < r^2 > \propto \text{ time since emission, t.} \tag{1}$$

The proportionality constant in Eq. (1) is photon diffusivity $D = c\ell_t/3$ with $\ell_t = \ell/(1-g)$ being the "transport" mean free path (MFP) in absence of absorption. Other notations are: c for the speed of light, ℓ = 1/extinction for the usual MFP, and $g = \langle \cos\theta_s \rangle = \langle \Omega \bullet \Omega' \rangle$ for the asymmetry factor of the scattering phase function $p(\Omega \to \Omega') \equiv p(\Omega \bullet \Omega')$, assumed azimuthally symmetric. When a numerical value is required, we will use the canonical value g = 0.85 for cloud droplets. In essence, the transport MFP ℓ_t is the MFP for one isotropic scattering—memory of beam's original direction is erased—which is the cumulative effect of $\approx 1/(1-g)$ forward-peaked scatterings on average (Davis and Marshak 1997a); for g = 0.85, this is about 7 scattering events.

Obtaining $\langle n \rangle_T \propto \tau^2$ from Eq. (1) is actually not totally straightforward. The trick here is to replace the mean-square-distance statistic by a given quantity and vice versa in $\langle r^2 \rangle = Dt$. Therefore, we make the following identifications:

- $\langle r^2 \rangle$ becomes H^2, where $H = \tau\ell$ is the given physical thickness of the medium.
- t becomes $\langle \Lambda \rangle_T/c$, where $\Lambda_T = ct \approx n_T\ell$ is the random pathlength covered by transmitted photons.

In summary, we find that the mean number of "effectively isotropic" scatterings, $\langle\Lambda\rangle_T/\ell_t \approx (1-g)\langle n\rangle_T$, is approximately equal to the rescaled optical depth, $(1-g)\tau$, squared. From there, we find

$$< n >_T \approx (1-g)\tau^2 \tag{2}$$

QED. Equivalently, we can write this as

$$< \Lambda >_T / H \approx (1-g)\tau \tag{3}$$

The numerical prefactor in Eqs. (2) and (3) would be 3 if all the substitutions were carried out properly but this is not to be taken too seriously since several rough approximations were made here; the argument is essentially dimensional.

A rigorous derivation (Davis and Marshak 2000a) uses the time-dependent one-dimensional (1-D) diffusion equation and proper boundary conditions on a homogeneous plane-parallel medium leads directly to Equation 3. This exercise justifies the above ansatz, but yields 1/2 rather than 3 for the prefactor. It also provides a pre-asymptotic correction term $[1+C_T(1;\varepsilon_\tau)]$ where $\varepsilon_\tau = 2\chi/(1-g)\tau$ (which, in diffusion theory, is also Transmittance/Reflectance in absence of absorption) and $C_T(1;\varepsilon_\tau) = (\varepsilon_\tau/2)(4+3\varepsilon_\tau)/(1+\varepsilon_\tau)$. The first argument of $C_T(1;\varepsilon_\tau)$ refers to the order of the statistical moment of L_T: 1 for the mean, 2 for the root-mean-square, etc. The numerical factor χ defines the "extrapolation length" $\chi\ell_t$ at the boundaries of the slab (e.g., Case and Zweifel 1967) and is best determined numerically; Monte Carlo simulations for optically thick slabs suggest $\chi \approx 0.57$. Note that $C_T(1;\varepsilon_\tau)$ vanishes algebraically for small ε (large τ) but is still unity for $\varepsilon_1 \approx 0.55$, i.e., at rescaled optical depth $(1-g)\tau_1 = 2\chi/\varepsilon_1 \approx 2.07$; this translates to $\tau_1 \approx 13.8$, which is typical of stratocumulus layers. Since much lower (as well as higher) values are not rare, this correction is important. For similar analytical considerations in reflection based on $\langle n\rangle_R$ and $\langle\Lambda\rangle_R$, with applications to passive and active remote sensing, we refer to Marshak et al. (1995), and Davis et al. (1997, 1999a). While these authors use standard diffusion theory and Monte Carlo simulations for conservative scattering, Platnick's (2000a) approach uses the radiative transfer equation to address transmission, reflection, and absorption; Davis and Marshak (2000b) also treat absorbing cases.

Anomalous Diffusion Theory

An even less acknowledged premise is that the optical medium has to be very nearly homogeneous, i.e., the MFP is almost the same everywhere. Along with

$$(1-g)\tau = H/\ell_t \geq 1 \tag{4}$$

this is precisely what makes the diffusion assumption reasonable in the first place. To generalize this type of analysis to highly variable media, Davis and Marshak (1997) proposed to use an alternative to Eq. (1) in continuous-time random walk theory where

$$\text{some measure of average distance} \propto \text{time}^{1/\alpha} \tag{5}$$

with $0 < \alpha < 2$. The new exponent α is called the "Lévy index", and instead of narrowly distributed (Gaussian or exponential) steps, we now have broad Lévy-stable step distributions with slow power-law decays. Because of the relatively frequent occurrence of very large jumps, Mandelbrot (1982) describes the associated random walk as a "Lévy flight." These substantial jumps model the radiative exchanges between broken clouds, different cloud layers, cloud-surface reflections, and so on. For an illustration, consider the radiative interactions that occur in the scene captured in Figure 1.

Figure 1. Typical complex cloud scene requiring 3-D radiative transfer theory at all scales. This analog picture was taken from the space shuttle near the terminator. Although the almost-grazing illumination does emphasize the 3-D structure, it is not particularly unusual.

Contrary to common belief, the relative thinness of the atmosphere (10s of km) does not justify the use of 1-D radiative transfer theory over general circulation model (GCM) grid-box scales (100s of km), only if the clouds themselves are reasonably plane-parallel at the smaller scales where the simplified computation is applied. The same remark applies for a linear "cloudy+clear" combination that is prescribed in radiative parameterizations currently used in GCMs.

Upon close examination, the reasoning used above to go from Eqs. (1), (2), and (3) assumes only that the MFP ℓ is finite; nothing is required of higher moments. This however puts a lower bound on α, namely, $\alpha > 1$ because moments of the new step distribution are divergent for orders $q \geq \alpha$ (Samorodnitsky and Taqqu 1994). With that proviso, we can still obtain the MFP from its definition in terms of H and τ ($\ell = H/\tau$), while g still enters in $\ell_t = \ell/(1-g)$ as shown by Davis and Marshak (1997). Using similar arguments as above, we now find $\langle\Lambda\rangle_T/\ell_\tau \approx (1-g)\langle n\rangle_T \approx [(1-g)\tau]^\alpha$ from Eq. (5) for the effective number of isotropic scatterings; from there, we obtain

$$\langle n\rangle_T \approx (1-g)^{\alpha-1}\bar{\tau}^{\alpha} \quad (6)$$

equivalently,

$$\langle\Lambda\rangle_T / H \approx [(1-g)\bar{\tau}]^{\alpha-1} \quad (7)$$

The limit $\alpha \to 2^-$ takes us back to the standard (Gaussian step) diffusion model in Eqs. (2) and (3) and, this time, we are leaving the prefactor in Eq. (7) to be determined empirically. Numerical simulations with g = 0.00, 0.85 (Oliver Funk, personal communication) yield a prefactor of $\approx$0.5 for $\alpha = 2$ (as predicted), $\approx$1.4 for $\alpha = 1.5$, and it increases strongly as $\alpha \to 1^+$. This is to be expected since the MFP itself diverges (logarithmically) at $\alpha = 1$.

We see that the alternative (Lévy-step)—often called "anomalous"—diffusion model in Eq. (7) with $1 < \alpha < 2$ yields systematically smaller mean pathlengths $\langle\Lambda\rangle_T$ for transmitted light at fixed H, g and τ as α decreases. Davis and Marshak (1997) obtained other relations for the Lévy-flight model, including the desirable prediction of reduced albedo with increasing variability (i.e., α decreases away from 2). Davis et al. (1999b) illustrate these and other predictions with numerical simulations and a selection of photon traces from Monte Carlo experiments in 1-D Lévy transport. To visualize 3-D simulations in Lévy transport, the interested reader can download QuickTime™ movies from the following URL: ftp://picard.iup.uni-heidelberg.de (courtesy of Dr. O. Funk).

We need to emphasize here that in a situation where standard diffusion theory applies ($\alpha \approx 2$), a single unbroken/dense cloud layer, H in Eq. (3) is the thickness of the layer. In contrast, when the Lévy model prevails ($\alpha < 2$), then H in Eq. (7) represents the full vertical extent of the cloud system. In such a system, the effective g value will of course vary from level to level: g = 0 in cloud/aerosol-free regions, $g \approx 0.85$ in liquid clouds, $g \approx 0.75$ in ice clouds and aerosol, referring only to conventional wisdom. However, the scattering is predominantly by cloud droplets so we will continue to use $g \approx 0.85$ when necessary.

Note also that we have introduced the notation "$\bar{\tau}$" in the key Eq. (7) for an appropriate area-average optical depth. There are indeed spatial considerations that come hand-in-hand with the above results in the time-domain and that are important for the following sections on observations. The question is:

> If light is released from a point at the boundary of a non-absorbing cloudy medium, over what area does it escape in transmission?

Here again the idea of an isotropically propagating "wave" of diffusing photons is helpful: if the medium's physical thickness is H, then we can expect the diffuse spot of light on the opposite (transmission) side to cover about an area H^2, and this is the region we need to average the optical depth over. This should not depend on whether the diffusion process is normal (Gaussian) or anomalous (Lévy), although the meaning and value given to H can be quite different in either application. At any rate, this prediction is well verified by a recent study of the radiative "smoothing" of cloud-transmitted solar radiance (Savigny et al. 1999) under conditions of heavy to total cloudiness. Reflection is another matter altogether and, for answers to the above question, we refer the reader to Marshak et al. (1995), Davis et al. (1997, 1999a), Platnick (2000b), and Davis and Marshak (2000b) for the basic analysis and applications to passive and active cloud remote sensing.

Model Motivation and Validation

Originally, the Lévy-flight photon transport model was proposed only to reproduce in a 1-D setting the important domain-average properties of massive (1024 × 1024) but simplified (4-beam) computations of radiative transfer in synthetic multifractal media (Davis 1992). Notwithstanding, Pfeilsticker (1999) presents compelling empirical evidence based on the high-resolution spectrometry of zenith radiance in the O_2 A-band that Eq. (7) has—thanks to α—enough flexibility to describe photon transport in the real cloudy atmosphere. For a re-analysis of the same data with an improved treatment of the forward scattering using g, we refer again to Davis et al. (1999b).

The FORTÉ Mission

FORTÉ is a small satellite experiment jointly designed, assembled, and operated by Los Alamos National Laboratory and Sandia National Laboratories. Its primary purpose is to address technology issues associated with nonproliferation treaty verification and the monitoring of nuclear tests from space. FORTÉ was launched on August 29, 1997, with an Orbital Pegasus and is now situated in a nearly circular, 70° inclination orbit at an altitude of approximately 825 km with a period of about 100 minutes. FORTÉ carries VHF broadband radio receivers (cf. Figure 2) as well as an Optical Lightning System (OLS). This instrumental package is optimized for the detection of lightning transients. The OLS is comprised of

- a broadband photo-diode detector (PDD) that collects records of lightning transients in a wide (80°) field-of-view
- a 128 × 128 pixel charge-coupled device (CCD) array called the Lightning Location System (LLS) using narrowband filter (777.6±0.5 nm) to isolate the strong OI(1) emission line in lightning.

The latter device provides imaging and geolocation of the transients to within a pixel size of 10 × 10 km^2. Together, the two sensors provide a high-resolution spatial and temporal description of detected lightning events. In this study, we use coincident data from FORTÉ's VHF receivers and the PDD component of the OLS, which are described in full detail respectively by Jacobson et al. (1999) and Kirkland et al. (1998); in the remainder of this section, we give brief summaries.

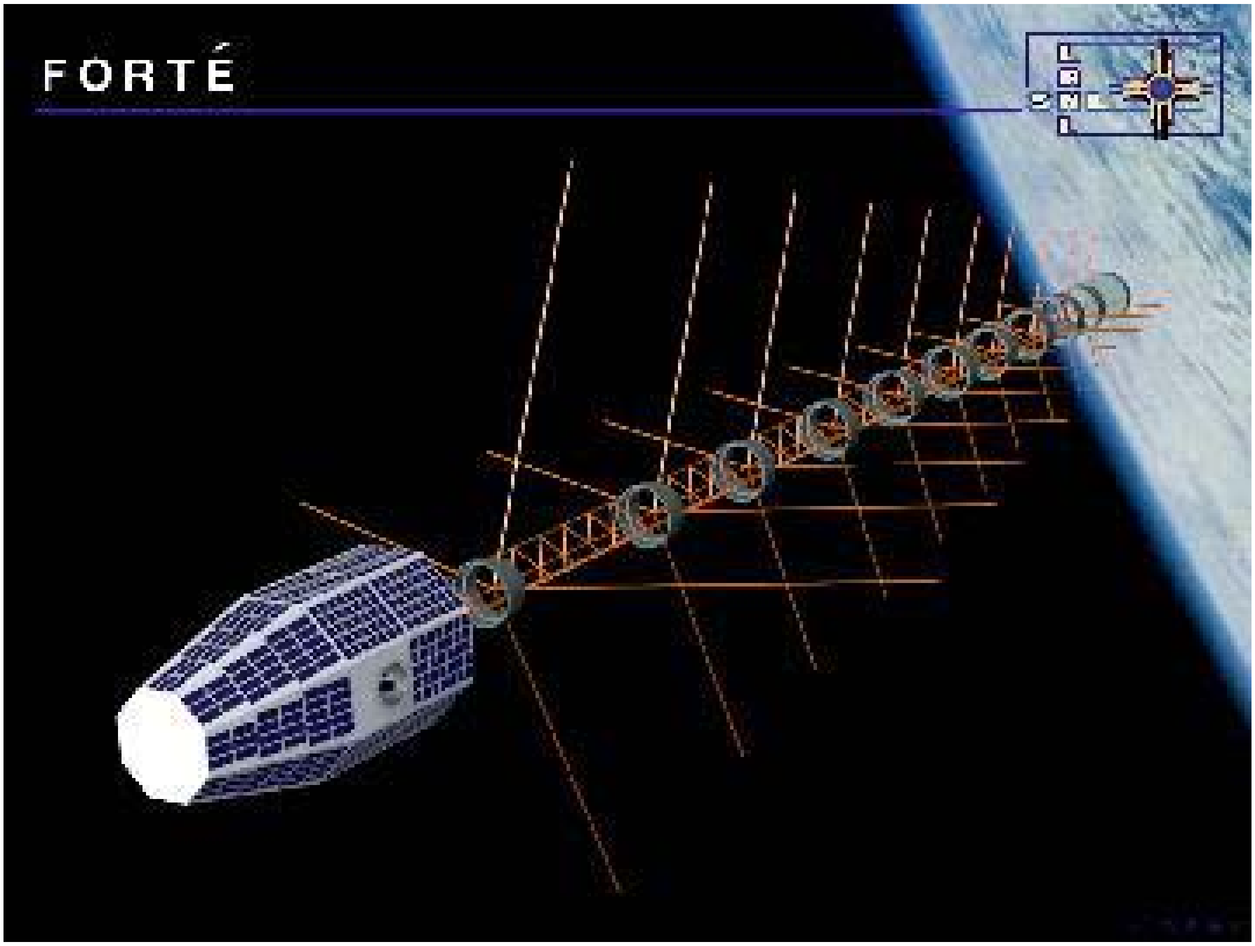


Figure 2. The Fast On-orbit TRansient Experiment (FORTÉ) mission. The VHF antenna is clearly seen in this artist's rendering. Also on board is the Optical Lightning System (OLS) consisting of a narrowband imager for transient geolocation and a broadband/wide-field-of-view (FOV) photo-diode detector for detailed waveform characterization.

The VHF instrumentation consists of two broadband receivers that can each be independently configured to cover a 22 MHz sub-band in the 30 MHz to 300 MHz frequency range, which is mostly populated by radio and television signals. For this study, one receiver was chosen to span the 26 MHz to 48 MHz range and the other spanned the 118 MHz to 140 MHz range. The instruments were configured to collect 40,960 samples in an 800 ms record length resulting in a time resolution of 20 ns (sample rate of 50 10^6 s^{-1}). The trigger point in each record allowed for 500 ms of pre-trigger information and 300 ms of post-trigger information. The record length and pre/post trigger intervals were chosen to optimize the detection and identification of the VHF lightning emissions. Data collection is triggered off the lower (26 MHz to 48 MHz) band receiver when the amplitude of its detected signal exceeds a preset noise-riding amplitude threshold in at least five of eight 1-MHz-wide sub-bands distributed throughout the 22-MHz bandwidth. This triggering technique allows the instrument to trigger on and detect weak lightning signatures in the presence of strong interfering manmade carriers. Re-triggering can occur after only a few microsecond delay, allowing the instrument to record extended multi-record signals with essentially zero dead-time. The FOV of the VHF receivers is determined by the antenna pattern; the 3-dB attenuation contour of the antenna response approximates a circle about 1200 km in diameter and was chosen to roughly correspond to the FOV of the PDD.

The PDD is a broadband (0.4 μm to 1.1 μm) silicon photo-diode detector that collects amplitude versus time waveforms of lightning transients. The instrument has an 80° FOV, which translates to a footprint of about 1200-km diameter from an 825-km altitude orbit. The instrument is typically configured to produce 1.92 ms records with 15 μs time resolution. The PDD is typically amplitude-threshold triggered, with a noise-riding threshold, and with a requirement that the signal exceed the amplitude threshold for five consecutive samples before a trigger occurs. This protocol eliminates false triggers due to energetic particles. However, the instrument can also be slaved to the VHF receivers whereby a trigger is forced whenever a VHF signal is received. The PDD provides 12-bit sampling with a piece-wise linear dynamic range covering four orders-of-magnitude and a sensitivity of better than 10^{-5} W/m^2. Several background compensation modes allow the instrument to be operated both at night and, at a reduced sensitivity, in the day. There is also a minimum inter-trigger delay of about 4.4 ms, which results in a ≈ 2.5 ms minimum dead time between successive records. The trigger times of both the VHF and PDD records are GPS time-stamped at 1-ms precision.

Statistics of Optical Waveforms from Lightning

We are interested here primarily in finding the signature of the 3-D variability of cloudiness structure on the propagation times for optical photons through the atmosphere, fully accounting for the delaying effect of multiple scattering. Therefore, that information needs to be extracted from the FORTÉ data record. We are mostly interested in the PDD responses to lightning transients, which are called optical "waveforms" even though it is radiant energy, rather than amplitude that is measured. However, the associated radio-frequency waveform is useful to characterize the lightning type and its inherent duration.

Sample FORTÉ data is shown in Figure 3 while the schematic in Figure 4a defines the relevant time delays between and inside the VHF and optical waveforms. In the overall trigger-to-peak delay Δt_{corr} from the radio to optical signals at the detector, we distinguish:

- the "physical" delay Δt_{phys} which is already present at the source and is thus traceable to plasma processes
- the "scattering" delay Δt_{scat} that describes how much the light pulse has been broadened/flattened during propagation from the source to the top of the atmosphere (TOA).

Figure 4b displays histograms of both Δt_{corr} and Δt_{scat} based on 237 well-characterized events. These events were carefully selected out of several thousand candidates that could be identified with National Lightning Detection Network (NLDN) ground-truth database. Suszcynski et al. (2000) describe in full detail the data processing and discuss in depth the underlying phenomenology of VHF and optical lightning signal production and propagation. In the remainder of this paper, we focus on the statistics of Δt_{scat}.

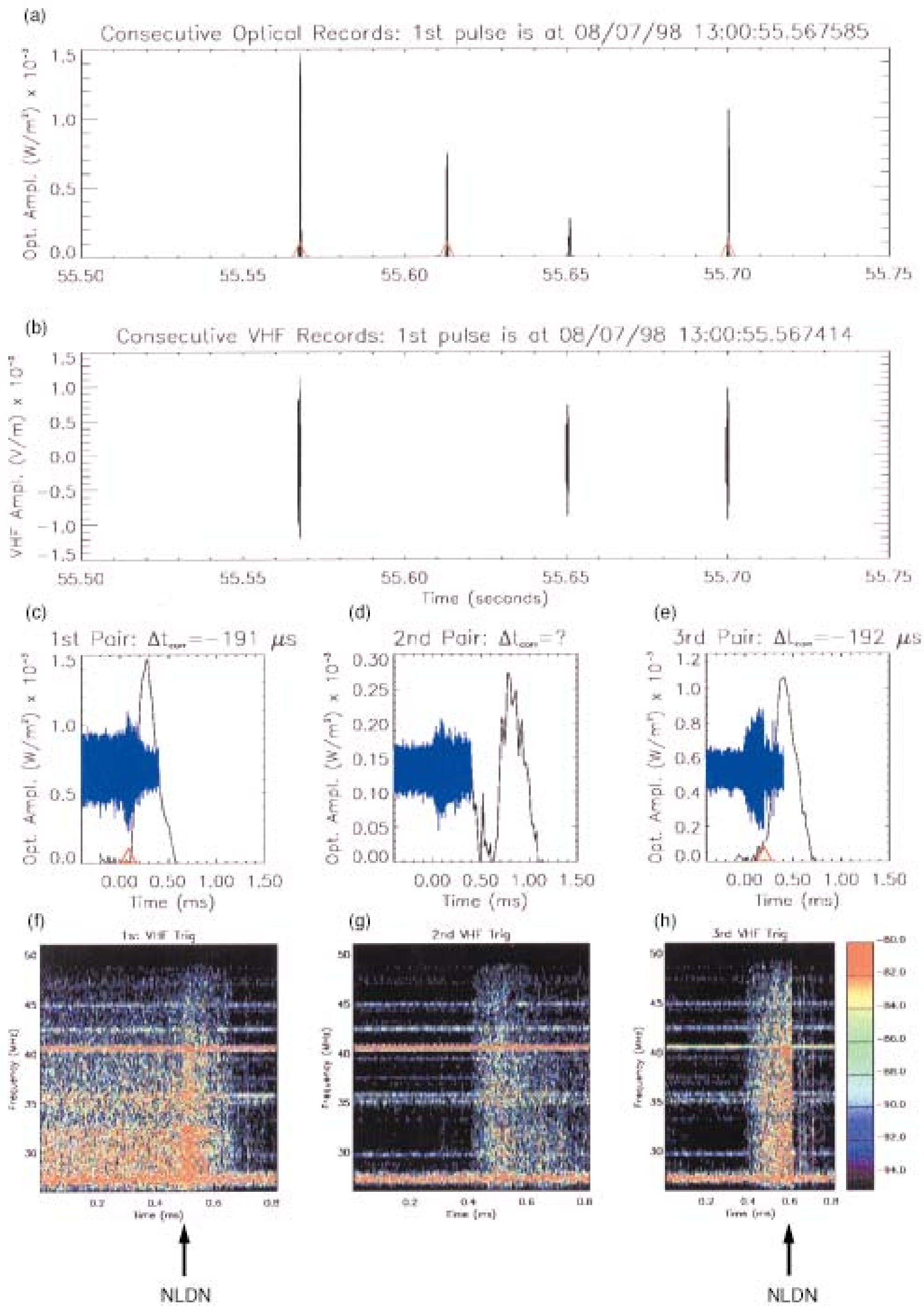
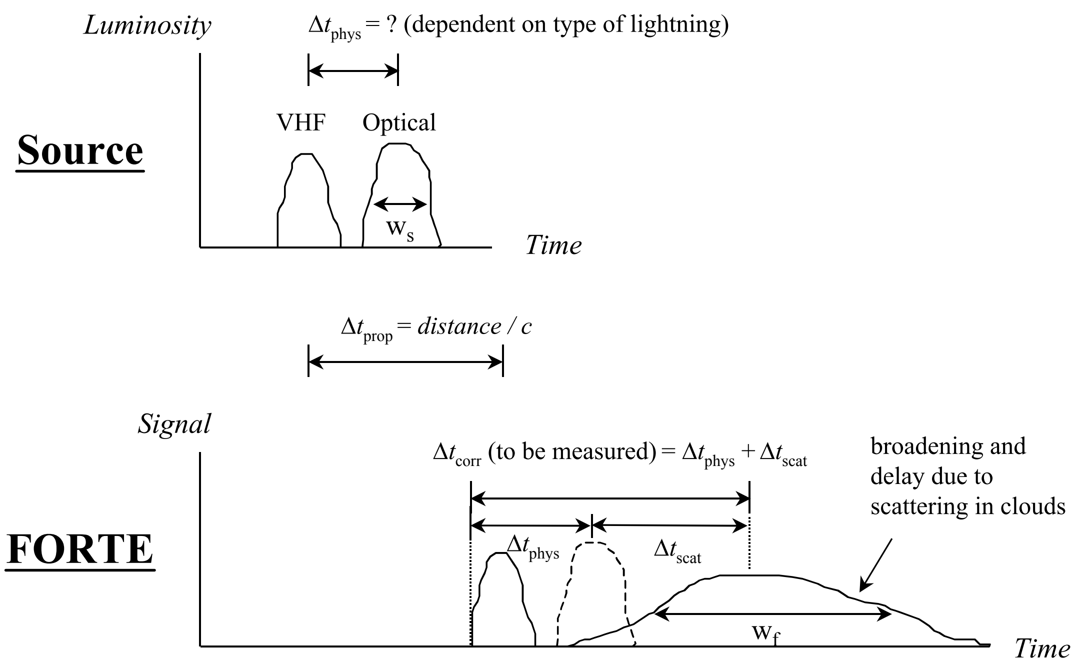
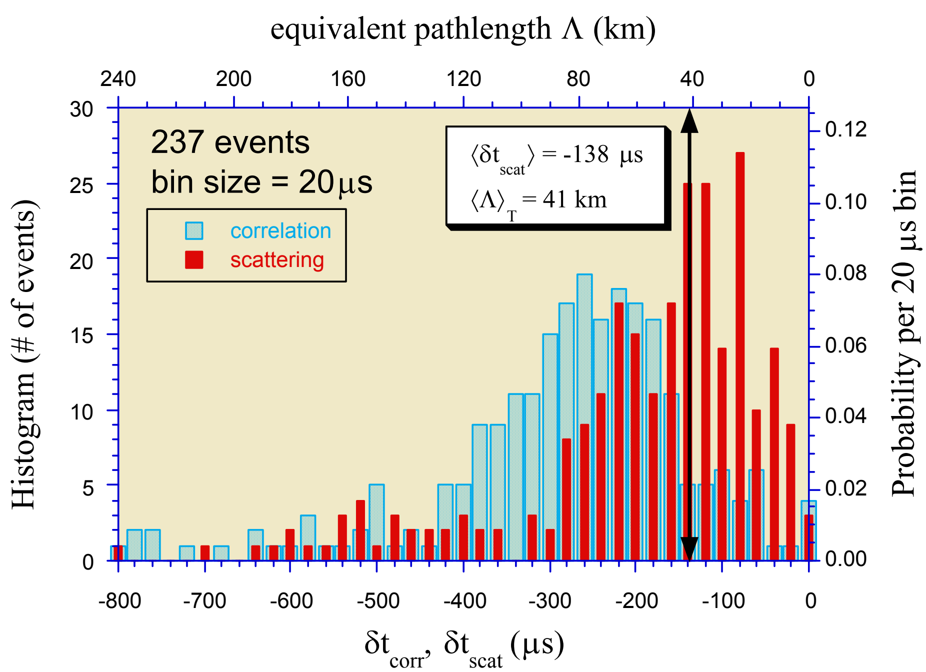


Figure 3. Typical FORTÉ data records. The top panel (a) shows 4 consecutive PDD records reconstituted to look like a time-series from the GPS time-stamps. The next panel (b) shows associated records in VHF signal that showed bursts and were captured for 3 cases. The remaining panels show close-ups of these coincident events. The two-dimensional (2-D) color plots at the bottom in panel's f to h are frequency-time representations of typical VHF transients and the associated 1-D line plots in panel's c to e are the radio- and optical-waveforms.



(a)



(b)

Figure 4. Lightning Waveform Phenomenology. (a) Schematic showing radio and optical emissions at the source, and how they appear to the distant observer in space. (b) Histograms showing relative frequencies for Δt_{corr} and Δt_{scat} (and the associated pathlength) in a population of 237 fully characterized events.

From Figure 4a it is clear that the "rise-time" (or trigger-to-peak) definition of Δt_{scat} is different from the mean scattering time $\langle\Lambda\rangle_T/c$ in Eqs. (3) and (7); the pulse width W_s, also defined in Figure 4a, is not directly related to $\langle\Lambda\rangle_T/c$, either. Our definition simply uses the waveform as a Probability Density Function (PDF) and, furthermore, we account for the minimum time H/c for light to cross the medium on a ballistic path. That is, before it is even theoretically detectable (by very unlikely direct transmission). Observed waveform-PDFs used here (cf. panels c and e in Figure 3) are somewhat positively skewed so $c\Delta t_{scat}$ is slightly smaller than $\langle\Lambda\rangle_T$. For the mode and mean of a PDF to differ significantly and systematically (beyond an O(1) proportionality factor), the waveform would have to be quite pathological, with multiple peaks and/or considerable skewness. This is not expected and, if observed, it is not used here (cf. panel d in Figure 3). Therefore, we proceed assuming that $\langle\Lambda\rangle_T \approx c\Delta t_{scat}$, given that there is already a similar uncertainty in the prefactor in Eq. (7) and that, unless $\alpha \approx 1$, $\langle\Lambda\rangle_T$ is significantly larger than the additive correction term $\approx$H, given Eq. 4.

Although scattering probability per collision (or single-scattering albedo ϖ_0) varies considerably over the shortwave spectrum, the transmitted signal is dominated by wavelengths with $\varpi_0 \approx 1$ (as weighted by the emission spectrum of lightning). As first-order approximation, the above analytical radiative transfer considerations, implicitly at $\varpi_0 = 1$, apply here.

Lévy Diagnostics from FORTÉ Data

Lévy Plots

We do not yet have the skill to determine the effective value of α needed to predict the radiative transfer for any given cloud distribution, beyond a single dense layer that yields $\alpha = 2$. Nonetheless, Pfeilsticker (1999) shows convincingly that his inferred values of α are properly stratified between 2 and 1 with respect to cloudiness: the more partial it is, the lower the α. Therefore, until we acquire this predictive skill, we consider the Lévy model as a diagnostic tool: radiative and available ancillary data (e.g., $\bar{\tau}$ and/or H) are used to infer the effective α, which, in turn, can be used as data to compare with other observables (e.g., cloud fraction).

Figure 5 is a diagnostic Lévy plot: $\langle\Lambda\rangle_T/H$ vs. $(1-g)\bar{\tau}$ in log axes graced with a few regression lines from Eq. (7) for given α values between 1 and 2 (using, for simplicity, a unit prefactor). Pfeilsticker (1999) and Davis et al. (1999b) use a slightly different Lévy plot based on $\langle\Lambda\rangle_T/\ell \approx \langle n\rangle_T$ vs. H/ℓ = as $\bar{\tau}$ suggested by Eq. (6). For our FORTÉ data on $\langle\Lambda\rangle_T$, we unfortunately do not have coincident values for $\bar{\tau}$ and H. However, we can make some reasonable assumptions. Since there is a strong causal relation between lightning and deep convection, we can safely assume H is several km, up to the full thickness of the troposphere. Turning to optical depth, the mean $\bar{\tau}$ over an area $\approx H^2$ for a convective storm system is likely to be in a range going from somewhat less than 100 to several 100. Convective cores can probably be even more opaque but, by definition, they only cover a fraction of the area of interest to the lightning photons escaping towards detection in space. So in Figure 5, we have defined the region of interest by drawing a few horizontal $\langle\Lambda\rangle_T/H$ = constant lines using the $\langle\Lambda\rangle_T$ data in Figure 4b. Specifically, we consider the 41-km mean of the 237-event sample, we also consider values

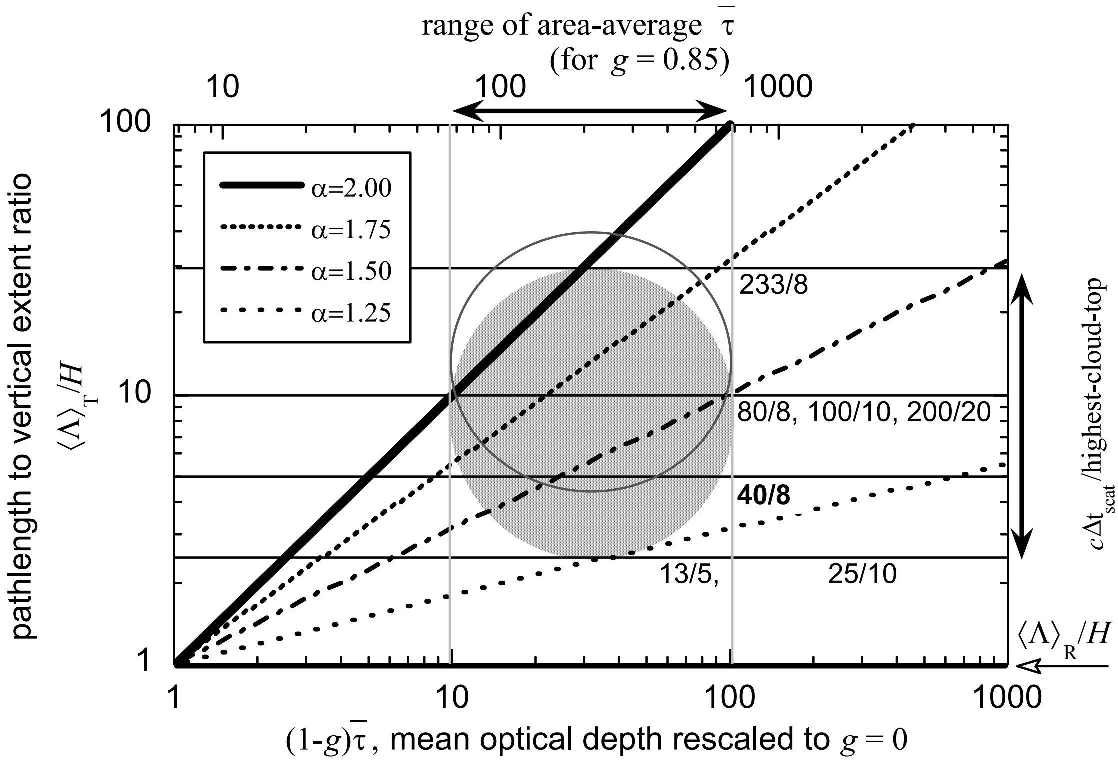


Figure 5. Likely locus of FORTÉ lighting data on Lévy plot. For the available data, we had no information on H, the overall thickness of the cloudy portion of the atmosphere, not to the area-mean optical thickness $\bar{\tau}$. Therefore, we made some reasonable assumptions, knowing that lightning is mostly produced over land in storm systems dominated by deep convection, essentially from ground to the tropopause. H is therefore equated with tropopause height, or somewhat less. We also estimated the range for $\bar{\tau}$. The lines in this log-log plot correspond to the transmission result in Eq. (7) while the horizontal axis maps to $\langle\Lambda\rangle_T$/H for α = 1 as well as to $\langle\Lambda\rangle_R$/H for reflection (Davis et al. 1999a). Further discussion in the main text.

significantly lower than the mean, 2×mean, and up to the max (all are stated in km). Then we normalize these numbers by different plausible values of H (also stated in km). Overall, the FORTÉ data clearly favors the Lévy model. Given the strong 3-D variability of cloud structures generated by electric storms, this is not a surprise. Quasi-ballistic propagation (α = 1) can confidently be excluded; this is not surprising either since Pfeilsticker (1999) and Davis et al. (1999b) mapped this limit to almost clear-sky situations.

Discussion

The time-dependent radiative transfer of lightning has been modeled numerically by Thomason and Krider (1982) using a Monte Carlo technique and by Koshak et al. (1994) using (standard) diffusion theory. In both studies the focus is implicitly on the convective core since the cloud model is typically modeled as a sphere or a cylinder of some kind, always internally homogeneous. Within the limited framework of homogeneous cloud models, the idea of using a horizontally infinite plane-parallel slab never comes to mind in lightning studies whereas it is still universally used in solar and thermal radiative transfer. Slab geometry is indeed justifiable for stratiform clouds (cirro-stratus, alto-stratus, nimbo-stratus, stratus, and even strato-cumulus), but not for towering convective clouds (cumulus congestus and cumulo-nimbus) in isolation from their environment. However, such convective clouds are not in an optical vacuum and the more comprehensive modeling approach advocated here is based on the assumption of plane-parallel geometry for the whole atmosphere but with strong internal variability.

Two questions remain:

1. How is it that Koshak et al.'s model can be considered as relatively successful in reproducing transmitted lighting waveform shapes while it is mapped to our $\alpha = 2$ case?

2. Why would a homogeneous sphere, cylinder or parallelepiped be good enough for lightning when a homogeneous plane-parallel slab is insufficient for solar and thermal applications?

To address the first question, we need to have a closer look at Figure 5. The extreme case of standard diffusion modeling ($\alpha \to 2^-$, Lévy $\to$ Gauss) is not excluded; rather, it seems unlikely to explain the bulk of the FORTÉ data. Furthermore, if we account for the additive and multiplicative biases between $\langle\Lambda\rangle_T$ and $c\Delta t_{scat}$, as discussed in the previous section, the gap then narrows slightly (we illustrate this in Figure 5 where the region-of-interest is nudged upward then flattened from below). On the other hand, no attempt was made to remove the effect of inherent source duration here. This makes the measured $\langle\Lambda\rangle_T$ longer than what the theory predicts for a δ-in-time source, by at least as much as the $\approx$H additive correction we just applied.

Although this is not the source/observer geometry used by Koshak et al. (1994) we note that slant viewing from above of a horizontally-finite cloud irradiated from below (by a cloud-to-ground stroke) combines the transmitted light discussed here along with light from "side-leakage." This side contribution follows the scaling for reflected light: $\langle\Lambda\rangle_R \sim H$, independent of g and τ in standard diffusion (Davis et al. 1999a) and probably in anomalous diffusion too. (Note that the definition of H may need to be modified for the finite horizontal extent to something like "mean distance from the source to the non-illuminated boundaries.") So $\langle\Lambda\rangle_R/H$ is along the horizontal axis in the (log-log) plot in Figure 5 while $\langle\Lambda\rangle_T/H$ is along the diagonal: a combination of the two can always be found to fit any FORTÉ data (even if it is linear rather than harmonic). The plane-parallel/anomalous-diffusion approach to the horizontal and vertical variability is however more natural.

The answer to the latter question is simply that lightning radiative transfer is only at its beginning both in theory and in observation. Therefore, the most simplistic models are (momentarily) good enough as soon as their parameters are tuned to reproduce the limited data. In contrast, solar/thermal radiation transport has already logged many decades of extensive research driven by climate modeling and remote sensing, although it is still grappling with some fundamental problems. To wit, consider the issue of absorption enhancement by clouds (Charlock et al. 1998, and references therein). In this case, when parameters are tuned to match for instance reflectance with data, then transmission and absorption are off at least for some of the well-documented cases. The reason for this is still a matter of speculation; as far as we know (from numerical simulations), 3-D radiative transfer effects can only be part of the solution (also recall from Eq. (7) that lower α values imply shorter pathlengths in a finite medium). Being inherently time-dependent, lightning studies are more of a challenge in modeling as well as in instrumentation. The FORTÉ mission is a major breakthrough in measurement technology and requires a sustained effort in theoretical modeling. We expect the Lévy photon-transport model will provide a partial response to this demand.

Conclusions, Outlooks, and Recommendations

Observation

We found new evidence in support of the Lévy/anomalous diffusion model as an accurate representation of photon transport in the cloudy atmosphere. This comes from carefully looking at the statistics of lightning waveforms captured in space by DOE's nonproliferation technology satellite FORTÉ. Unfortunately, only time-domain observations of the photon pathlength $\langle\Lambda\rangle_T$ in transmission were available so some educated guesses had to be made about other key quantities such as area-average optical depth $\bar{\tau}$ and overall thickness of the cloud system H. However, since the light-source here is lightning, the variability of these parameters is limited. Nonetheless, future work should use correlative data for $\bar{\tau}$ and H which in principle is obtainable though FORTÉ's Lightning Location System.

The first empirical study of the Lévy transport model used O_2 A-band spectroscopy at very high resolution (Pfeilsticker 1999). It would be worthwhile to see where Min and Harrison's (1999) pathlength observations fall in Lévy plots. This data was also harvested from the A-band but at the moderate resolution of an autonomous ARM instrument, the rotating shadowband spectroradiometer (RSS) while optical depths were obtained from the collocated passive microwave radiometer (MWR). We are of course looking forward to A-band measurements from space that will, in due time, be collected by the NASA/ESA mission PICASSO/CENA as well as by NASA's CloudSat mission. At present, simulations by Stephens and Heidinger (2000) based on Cahalan's (1989) fractal cloud models tell us that the signature of 3-D variability is an increase in mean pathlength $\langle\Lambda\rangle_R$ for reflection. Interestingly, this is the opposite trend of what we find here for transmission in Eq. (7) for $\langle\Lambda\rangle_T$ but similar to numerical results for a milder variant of the fractal clouds (Marshak et al. 1995; Davis et al. 1997). This 3-D effect is also compatible with a Jensen-inequality argument applied to the concavity with respect to τ (the most variable parameter in clouds) of the significant correction term found by Davis et al. (1999a) for $\langle\Lambda\rangle_R$. Specifically, they find

$$< \Lambda >_R = 2\chi H \times [1 + C_R(1;\varepsilon_\tau)] \tag{8}$$

where $\varepsilon_\tau = 2\chi/(1-g)\tau$ is ≈ 1 or greater and $C_R(1;\varepsilon_\tau) = (\varepsilon_\tau/2)(1+3\varepsilon_\tau/2)/(1+\varepsilon_\tau)$; see the section From Photon Diffusion to Lévy-Flight Heruistics for the meaning and the O(1) value of χ. At any rate, the opposite effects of the unresolved variability on albedo and mean pathlength will enable a joint retrieval of mean optical depth (not just the "effective" optical depth) and its variance. The latter quantity is badly needed to validate GCM physics schemes with sub-gridscale variability parameterizations.

Modeling

The Lévy-diffusion transport model has the flavor of a GCM parameterization, a 1-D computation that somehow incorporates unresolved (i.e., sub-gridscale) structures. However, we do not know yet how to relate given variability to the variability parameter "α." Furthermore, the Lévy-index α is currently assigned to the whole column rather than a single layer. Currently, a considerable amount of effort is being invested in a promising approach based on Cahalan's (1989) independent-column (or -pixel) assumption. For instance, Barker's (1996) model is predicated on Gamma-distributed optical depths populating the columns and an otherwise standard 2-stream approximation in each one. In this approach, it is important to determine to what extent 3-D effects (beyond the independent-column assumption) cancel each other in the domain-average, e.g., shadowing versus illumination of cloud sides. Evans and Benner (1999) show that these specific 3-D effects induced by the sun-angle almost cancel over the diurnal cycle, if not instantaneously. The appeal of a GCM parameterization based on Lévy-diffusion is that it comes pre-validated with vastly different observations: lightning photons in the time-domain, solar photons in A-band absorption features (effectively the Laplace conjugate of time).

As a 1-D surrogate for 3-D radiative transfer, the Lévy-diffusion model for the moment lacks an analytical formulation (using differential or integral equations), which, in turn, is a prerequisite to develop an effective GCM parameterization. Bäumer et al. (2000) devised a rigorous formalism based on fractional derivatives that translates solutions of the standard (integer-order) diffusion equation into solutions of fractional differential equations that describe Lévy processes in sub-surface hydrology. We hope this methodology can be adapted to our concerns in atmospheric radiation transport. At any rate, it is clear that Lévy-transport, when framed as an integral equation, will have a kernel with power-law rather than exponential decay (Hélène Frisch, personal communication). This kernel is closely related to the free-path distribution. So, on a more fundamental level, the successes of the Lévy model in explaining empirical facts about radiation transport in cloudy atmospheric columns re-open interesting questions about the relevance of Beer's exponential law of extinction. As far as we know, non-exponential photon free-path distributions were first discussed by Romanova (1975) in connection with early 3-D radiative transfer computations. Since then, the idea is implicit in a number of studies but Davis (1992) and more recently Kostinski (2000) address the problem directly.

Outreach

Turning to the emerging research field of lightning radiative transfer, we hope that the Lévy-diffusion model presented here will provide a useful alternative to current approaches; these are based on Monte Carlo simulation or explicit solutions of the standard diffusion in necessarily simple geometries. To quote Mandelbrot's (1982) classic invocation of fractal geometry: *'mountains are not cones, clouds are not spheres, [and so on ...].'* If we follow the historical development of solar/thermal radiative transfer over the last decade or so, a flexible Monte Carlo scheme is a key asset to improved lightning waveform modeling. Light et al. (2000) designed such code, offering a sustained numerical counterpoint to the analytic theme presented here.

In our opinion, a closer interaction between the radiation communities involved in solar/climate studies and lightning/nonproliferation monitoring can be mutually beneficial, this paper being only a beginning. One such interaction could be the use of lightning monitoring on the synoptic scale to better understand the possible link between extreme weather (shifting patterns, increased frequency, etc.) and climate change (global warming, cloud-mediated feedbacks, etc.). The livelihood and health of whole populations are threatened both by the acute problems caused by extreme weather events and by the trends we see in global change, if left unmitigated.

Acknowledgments

This work is supported by the U.S. Department of Energy. Specifically, AD and AM are funded by DOE's Office of Biological and Environmental Research of as part of the Atmospheric Radiation Measurement (ARM) Program. We also thank Drs. Larry Auer, Howard Barker, Boris Bäumer, Albert Benassi, David Benson, Bob Cahalan, Hélène and Uriel Frisch, Oliver Funk, Lee Harrison, Abe Jacobson, Matt Kirkland, Alex Kostinski, Tess Light, Shaun Lovejoy, Mark Meerschaert, Qi-Long Min, Klaus Pfeilsticker, Steve Platnick, Christian v. Savigny, Warren Wiscombe, and others for many stimulating discussions about Lévy/anomalous-diffusion in the atmosphere.

Corresponding Author

A. B. Davis, adavis@lanl.gov, (505) 665 6577.

References

Barker, H. W., 1996: A parameterization for computing grid-averaged solar fluxes for inhomogeneous marine boundary layer clouds – Part 1: Methodology and homogeneous biases. *J. Atmos. Sci.*, **53**, 2289-2303.

Bäumer, B., D. A. Benson, M. M. Meerschaert, and S. W. Wheatcraft, 2000: Subordinated advection-dispersion equation for contaminant transport. *Water Resour. Res.,* submitted.

Cahalan, R. F., 1989: Overview of fractal clouds. In *Advances in Remote Sensing Retrieval Methods*, eds., A. Deepak, H. E. Fleming, and J. S. Theon, Deepak Pub., 371–388.

Case, K. M., and P. F. Zweifel, 1967: *Linear Transport Theory*, Addison–Wesley, Reading, Massachusetts.

Charlock, T. P., F. G. Rose, T. L. Alberta, and G. D. Considine, 1998: Comparison of computed and measured cloudy-sky shortwave (SW) in the ARM Enhanced Shortwave Experiment (ARESE). In *Proceedings of the Eighth ARM Science Team Meeting*, U.S. Department of Energy, Washington, D.C., pp. 133–140. Available URL: http://www.arm.gov/docs/documents/technical/conf_9803/charlock-98.pdf

Davis, A. B., 1992: *Radiation transport in scale-invariant optical media*, Ph.D Thesis, McGill Un., Physics Department, Montreal, Quebec, Canada.

Davis, A. B., and A. Marshak, 1997: Lévy kinetics in slab geometry: Scaling of transmission probability. In *Fractal Frontiers*, eds., M. M. Novak and T. G. Dewey, pp. 63-72, World Scientific, Singapore.

Davis, A. B., A. Marshak, R. F. Cahalan, and W. J. Wiscombe, 1997: The LANDSAT scale-break in stratocumulus as a three-dimensional radiative transfer effect, implications for cloud remote sensing. *J. Atmos. Sci.*, **54**, 241–260.

Davis, A. B., R. F. Cahalan, J. D. Spinhirne, M. J. McGill, and S. P. Love, 1999a: Off-beam lidar: An emerging technique in cloud remote sensing based on radiative green-function theory in the diffusion domain. *Phys. Chem. Earth (B)*, **24**, 757–765.

Davis, A. B., A. Marshak, and K. P. Pfeilsticker, 1999b: Anomalous/Lévy photon diffusion theory: toward a new parameterization of shortwave transport in cloudy columns. In *Proceedings of the Ninth ARM Science Team Meeting*, U.S. Department of Energy, Washington, D.C. Available URL: http://www.arm.gov/docs/documents/technical/conf_9903/davis-99.pdf

Davis, A. B., and A. Marshak, 2000a: Space-time characteristics of light transmitted by dense clouds. *J. Atmos. Sci.* submitted.

Davis, A. B., and A. Marshak, 2000b: Multiple Scattering in Clouds: Insights From Three-Dimensional Diffusion/P_1 Theory, *Nucl. Sci. Eng.*, Special Issue “In Memory of Gerald C. Pomraning,” in press.

Evans, K. F., and T. C. Benner, 1999: Three-dimensional broadband solar radiative transfer in small tropical cumulus fields derived from high-resolution imagery. In *Proceedings of the Ninth ARM Science Team Meeting*, U.S. Department of Energy. Available URL: http://www.arm.gov/docs/documents/technical/conf_9903/evans-99.pdf

Jacobson, A. R., S. O. Knox, R. Franz, and D. C. Enemark, 1999: FORTÉ observations of lightning radio-frequency signatures: capabilities and basic results. *J. Radio Sci.*, **34**, 337-354.

Kirkland, M. W., D. M. Suszcynski, R. C. Franz, J. L. L. Guillen, and J. L. Green, 1998: *Observations of terrestrial lightning at optical wavelengths by the photodiode detector on the FORTÉ Satellite.* Rep. LA-UR-98-4098, Los Alamos National Laboratory, Los Alamos, New Mexico.

Koshak, W. J., R. J. Solakiewicz, D. D. Phanord, and R. J. Blakeslee, 1994: Diffusion model for lightning radiative transfer. *J. Geophys. Res.*, **99**, 14361–14371.

Kostinski, A. B., 2000: On the extinction of radiation by a homogeneous but spatially correlated random medium. *J. Opt. Soc. Am. A.,* submitted.

Light, T. E., D. M. Suszcynski, M. W. Kirkland, and A. R. Jacobson, 2000: Simulations of lightning optical waveforms as seen through clouds by satellites. *J. Geophys. Res.,* submitted.

Mandelbrot, B. B., 1982: *The fractal geometry of nature*, 460. W. H. Freeman, San Francisco, California.

Marshak, A., A. Davis, W. Wiscombe, and R. Cahalan, 1995: Radiative smoothing in fractal clouds. *J. Geophys. Res.*, **100**, 26247–26261.

Min, Q.-L., and L. C. Harrison, 1999: Joint statistics of photon pathlength and cloud optical depth. *Geophys. Res. Lett*., **26**,1425-1428.

Pfeilsticker, K., 1999: First geometrical pathlength distribution measurements of skylight using the oxygen A–band absorption technique—Part II: Derivation of the Lévy-index for skylight transmitted by mid-latitude clouds. *J. Geophys. Res*., **104**, 4101-4116.

Platnick, S., 2000a: A superposition technique for deriving photon scattering statistics in plane-parallel cloudy atmospheres. *J. Quant. Spectrosc. Radiat. Transfer*, **68**, 57-73.

Platnick, S., 2000b: Approximations for horizontal photon transport in cloud remote sensing problems. *J. Quant. Spectrosc. Radiat. Transfer*, **68**, 75-99.

Romanova, L. M., 1975: Radiative transfer in a horizontally inhomogeneous scattering medium. *Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, **11**, 509–513.

Samorodnitsky, G., and M. S. Taqqu, 1994: *Stable non-Gaussian random processes.* Chapman and Hall, New York.

Savigny, C., O. Funk, U. Platt, and K. Pfeilsticker, 1999: Radiative smoothing in zenith-scattered skylight transmitted through clouds to the ground. *Geophys. Res. Lett.*, **26**, 2949–2952.

Stephens, G. L., and A. Heidinger, 2000: Molecular line absorption in a scattering atmosphere - Part I: Theory. *J. Atmos. Sci*., **57**, 1599–1614.

Suszcynski, D. M., M. W. Kirkland, A. R. Jacobson, R. C. Franz, S. O. Knox, J.L.L. Guillen, and J. L. Green, 2000: FORTÉ observations of simultaneous VHF and optical emissions from lightning: basic phenomenology. *J. Geophys. Res.*, **105**, 2191-2201.

Thomason, L. W., and E. P. Krider, 1982: Effects of clouds on the light produced by lightning. *J. Atmos. Sci*., **39**, 2051–2065.